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NAVAL POSTGRADUATE SCHOOL

Monterey, California

CONTINUOUS AUTOROUTERS, WITH AN APPLICATION TO SUBMARINES

Alan Washburn

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Control theory is applied to the problem of routing a vehicle from one point to another in fixed time. The method requires an in itial guess at the route, which is then gradually warped into a route that is locally optimal. Application is made to a problem where a submarine wishes to find a route along which minimal radiated noise will be intercepted by the enemy.

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1. INTRODUCTION

Certain tactical problems take the form of inquiries into the best way of getting from A to B, where A and B are positions in a continuous state space. The optimal route from A to B is not necessarily a straight line: ocean currents or winds may cause a ship to be routed indirectly to take advantage of favorable areas, or certain regions may be threatening (typhoons, enemy units) or even non-feasible (land). A "route" being a complicated mathematical object, it should be expected that the time required for computation of an optimal route will be significant, and that it will be sensitive to the way in which the optimal routing problem is formulated and solved. This technical report describes a somewhat unconventional approach to formulation and solution. It includes a program demonstrating the technique in a problem where a submarine is to be routed past several listeners trying to detect it.

2. DISCRETE DYNAMIC PROGRAMMING VERSUS CONTROL THEORY

We will take the objective to be

$$\text{minimize} \int_0^T f\big(x(t), z(t), t\big)dt \tag{1}$$

subject to some constraints on the control variables z(t). x(t) for $0 \le t \le T$ is the route to be optimized, generally a vector. For brevity the arguments of x and z will be omitted below, but each is nonetheless a function of time (t). f(x,z,t) is *the rate at which penalty is accrued at time t*. For example f(x,z,t) might be *fuel used per unit time* if the problem were to minimize fuel usage or *rate of being hit by AAA* if the problem were to navigate an aircraft through defenses

with as little risk as possible. The control variables z influence the state x through the equation

$$dx / dt = g(x,z,t), \tag{2}$$

where the function g (like the function f) is given. The simplest case is where $g(x,z,t) = z$, in which case z can be interpreted as the controllable velocity corresponding to x. This case is also the part of Control Theory that corresponds to the Calculus of Variations.

The important feature of (1) is that it is an accumulation over time. This fact is exploited in both discrete Dynamic Programming (DDP) and Control Theory, the two commonly applied methods of solution. In DDP, the state space is made discrete and the minimization is performed by exhaustion; DDP owes its power to a clever ordering of the required minimizations, rather than to any exploitation of analytic properties of f or g. DDP produces global optima, an advantage, but it suffers from Bellman's *curse of dimensionality*—the number of state variables and the coarseness with which each is measured must be carefully controlled lest solution times rapidly become large.

Control Theory does not require that state variables be measured coarsely. In fact, there would be nothing illogical about using double precision arithmetic in manipulating them. Control Theory suffers instead from having to solve two-point-boundary-value problems. Roughly speaking, in applying Control Theory one easily obtains an optimal solution, only to discover that the wrong problem has been solved. The main computational effort comes in manipulating the wrong problem into the right one. Even when an optimal solution to the right problem is obtained, it may only be a

local optimum. Control Theory thus has its own difficulties, different from those of DDP but equally serious.

It should be mentioned that Dynamic Programming is actually the more general of the two techniques, since the functional equation of (continuous) DP can be used to derive the necessary conditions of Control Theory (Bellman and Dreyfuss [1962], Jacobson and Mayne [1970]). Bellman's Principle of Optimality appears to be the fundamental observation: if the optimal route from A to B passes through C, then the parts from A to C and from C to B must also be optimal for their respective problems.

The gist of the preceding paragraphs is that one can either begin by imposing some sort of grid on the state space, in which case DDP is the natural optimization technique, or one can begin by attempting to exploit analytic properties of f and g, in which case the necessary conditions of Control Theory are the natural result. If one chooses the latter, one must be prepared for the possibility that optima may be local, rather than global.

Most current tactical decision aids employ DDP in route optimization. Klapp [1979] describes the approach of the Fleet Numerical Oceanography Center in routing ships, essentially the imposition of a network of grid points on the ocean. The Naval Oceanographic and Atmospheric Research Laboratory also plans to incorporate a tactical environmental ship routing (TESR) function as part of the Tactical Environmental Support System. Weissinger [1987], describes an approach to TESR wherein states are positions in space-time. He estimates computation times from .25 hours to 15.8 hours, depending on how coarse the grid is, on a HP9020 microcomputer. The U.S. Air Force also incorporates some path optimization within its Mission

Planning System software. Jones [1986] describes how the WARPATH algorithm computes an optimal route, and discusses alternatives. All of these tactical route optimization programs find the shortest route through a network, with the node and arc costs depending on the application (Deo and Pang [1984]). The coarseness of the network is a crucial consideration in determining run times.

The coarseness inherent in the DDP approach suggests that some tactical routing problems might be better founded on Control Theory. Construction of such a prototype is the goal of the rest of this report. The technique is somewhat novel and fully described in the Appendix. The main idea is to eliminate the two-point-boundary-value problem, even at the cost of temporarily producing solutions that are non-optimal. The technique is roughly **steepest-descent**—a given route is gradually warped into something optimal by making first-order corrections, with the current route being at all times feasible. The initial route is a user input. The technique is employed to solve a problem where a submarine attempts to go from A to B without being detected, the objective being to minimize *total radiated energy received by enemy listeners*. The application and the prototype software are described in the next section.

As mentioned above, one of the weaknesses of Control Theory is that optima may be local, rather than global. This is of particular concern if there is reason to believe that the tactical problem is likely to have multiple optima. This will be the case, for example, in problems where a vehicle is to be routed past obstacles, since an "obstacle on the left" path cannot be warped into an "obstacle on the right" path without passing over the obstacle. The more

obstacles, the more local optima must be expected. In such problems, Control Theory will be a useful approach only if there is some other reliable mechanism for selecting a good starting point; i.e., a starting route that is likely to lead to a route that is globally as well as locally optimal. In two dimensional problems this mechanism may very well be the human user of the decision aid, since humans are good at seeing relationships in two dimensions. The division of effort would be that the human deals with topological issues, while the decision aid (via Control Theory) deals with detailed questions about direction and possibly speed. This is the approach taken in JITTER, the program to be described in the next section. The problem is first presented graphically to the user in such a manner that a reasonable route can be selected. JITTER then assumes a starting point where the speed along the input route is constant, eventually warping it into something that is locally optimal, possibly with variable speed.

3. SUBMARINE TRANSITS AND THE JITTER PROGRAM

Submarines radiate acoustic noise, with the amount of noise power radiated being a strong function of submarine speed. In trying to get from A to B without being detected by an enemy listener at C, a submarine may be tempted to steer far away from C, but in doing so may be forced to go so fast (time being constrained) that a remote detection by C may occur anyway. If there are actually several enemy listeners, the best track can be expected to be sinuous, but not so sinuous that its length forces extremely high speeds. Determination of the best track in these circumstances is a rather subtle problem, one that might reasonably be aided by a computer. This is the goal of the prototype program JITTER.

We will take the objective to be minimization of the total energy received by all listeners. It could be reasonably argued that a better criterion would be minimization of the maximum power received at any time by any listener. The truth is somewhere in between (Boyd [1989]), but there is little choice of criterion if Control Theory is to be easily applied—the criterion must be total received energy.

If there are n listeners, the power received by all of them at time t will be taken to be

$$p(t)=\sum_{i=1}^{n}\left\{s_i / r_i(t)^2\right\}\left\{1+\alpha v(t)^4\right\}\left\{1-\beta\cos\left(2\theta_i(t)\right)\right\}, \tag{3}$$

where (see Figure 1):

$r_i(t)$ = distance to listener i

$v(t)$ = submarine speed

$\theta_i(t)$ = bearing to listener i, relative to the submarine's bow

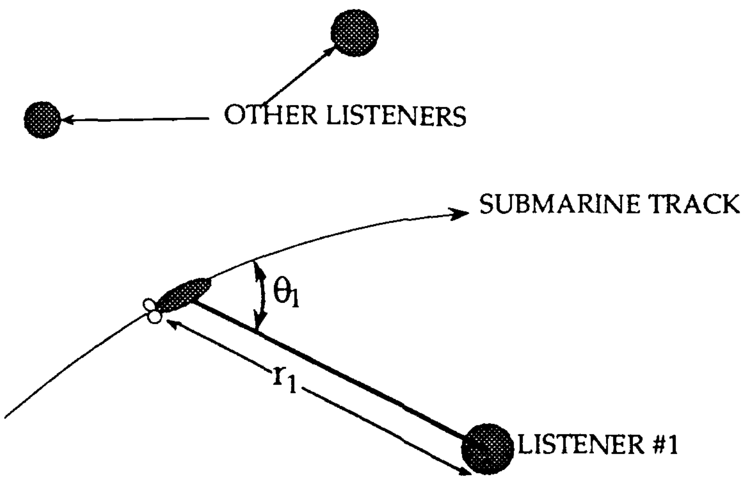

Figure 1. A Submarine Passing through Three Listeners

The three factors in { } in (3) can be thought of as **basic radiated power** followed by two correction factors. Division by $r_i(t)^2$ in the first factor

represents the assumption of spherical spreading; effects of reflections and refraction are being ignored. The constant s_i will depend on i if some listeners are more sensitive than others. The power required to drive a submarine increases roughly with the cube of speed. A small but increasing fraction of this is radiated as speed increases, so v(t) is raised to the fourth power in the speed correction factor (an ulterior motive here is that raising something to the fourth power doesn't require an exponentiation). The parameter α is set to .0003 in JITTER, which corresponds to doubling the radiated power, relative to small speeds, when the speed is 7.6 knots. The third correction factor is needed because power is usually not radiated equally strongly in all directions. The parameter β must be such that $|\beta| \leq 1$. When $\beta > 0$, power tends to be radiated most strongly broadside. β is arbitrarily set to .5 in JITTER, which corresponds to broadside radiation being three times as strong as bow/stern radiation. The submarine is assumed to be oriented in the same direction as its velocity vector. The model sketched above is correct in its tendencies for a non-cavitating submarine, although the parameters would of course need to be adjusted in a real application. A more detailed description of radiated noise can be found in Ross [1976].

P(t) as given by (3) depends on submarine position x(t) (needed to compute $r_i(t)$) and velocity z(t) (needed to compute v(t) and $\theta_i(t)$), so the path optimization problem is in the class described in Section 2. JITTER uses the optimization method described in the Appendix to solve it. The step size is dynamic; as long as the forecast improvement agrees with the actual improvement, JITTER will gradually increase it. If the forecast and actual improvements disagree sufficiently, the step size is reduced. The current

route and the objective function are continuously displayed, and the user is invited to stop the program whenever convergence has occurred, possibly restarting it on a different route that connects the same two points. The reader is invited to try using JITTER, which is included on the attached diskette.

One of the more interesting features of JITTER solutions is the presence of "kinks" (discontinuities in velocity) on the optimal route in problems where the input average speed is low (5 knots, say). These kinks are at first surprising but actually reasonable, since:

1) Velocity is the control variable, and optimal control problems often involve discontinuities in control variables.

2) The submarine would like to avoid exposing its sides to close listeners because $\beta > 0$. Therefore sharp turns through significant angles and bow-on approaches to obstacles should be expected.

Real submarines can't make sharp turns, so any proferred "solution" with sharp turns in it is of course only a rough guide to what should actually be done. Kinks disappear when the input average speed is high (20 kt, say). The kinking problem could be avoided by making velocity a state variable and letting the control variables be (say) rudder angle and acceleration.

To run JITTER, an MS-DOS computer with either an EGA or VGA graphics board is required; the program tests for the right hardware and will terminate if conditions aren't satisfactory. The 8087 chip is not required, nor is a color display (but by all means choose a color display if one is available). JITTER first reads the chart scale and listener locations from the file SITES.DAT that is included on the diskette (along with a file SITES.EXE that can be used for changing SITES.DAT if desired). JITTER echoes this data to

the screen, asks for an average speed input, and then produces a map on which the user inputs a candidate route in a *connect the dots* fashion. The solid disks on the map represent the listeners, with the area of each disk being proportional to the listener's sensitivity (there is no special significance if a submarine track penetrates one of these disks). Once calculations start, the current route is displayed using line segments of alternating color, each of which corresponds to a fixed amount of time, so an impression of speed can be got from the lengths of the segments. Calculations will be interrupted when JITTER senses that no further improvement is possible, in which case the user can either terminate the program or input another candidate route that connects the same initial and terminal segments. In the latter case, JITTER displays the smallest measure of effectiveness (received energy) that has been achieved by previous tries.

It should be mentioned that decisions aids like JITTER could deal with moving listeners, as long as the track that each listener follows is known. The main complication would be graphical, rather than mathematical, since it would become difficult to display the listener tracks while asking the user to input a reasonable starting solution.

Other points worth noting about JITTER

1) The vertical dimension reads downwards! This is the author's revenge on submariners, who are forever plotting transmission loss curves in that manner.

2) The most likely reason for a *division by zero* termination is a leg of zero length. The easiest way to do this is to fail to move the cursor between inserts.

3) Don't route the submarine directly over one of the listeners. If you do, JITTER will respond by lengthening the segment that passes over

the listener, rather than avoiding the listener. This action "works" because distances are only evaluated at the ends of line segments; the phenomenon could be avoided by increasing the number of line segments, but doing so would slow the program down.

4) The user's first input will be "average speed as the crow flies," this input being used only to calculate the amount of time available. The actual average speed of the submarine on the initial track will be greater than this to the extent that the track is longer than the shortest distance between A and B.

5) The source code in Turbo Pascal is available from the author (408-646-3127).

APPENDIX

Consider the problem of minimizing

$$\int_0^T f(x, dx/dt, t)dt \qquad (A1)$$

where x(0) and x(T) are given. x and dx/dt depend on t, but this dependence on time is suppressed in the notation. This is the classical fixed end point problem of the Calculus of Variations. We first approximate (A1) by a sum. Let $T = N\delta$, where N is a large integer, let $x_i = x(i\delta)$; $i = 0, \ldots, N$, and let $\dot{x}_i = (x_i - x_{i-1})/\delta$, $i = 1, \ldots, N$. Then the problem is to minimize

$$J(0) = \delta \sum_{i=1}^{N} f(x_i, \dot{x}_i, i\delta) \qquad (A2)^*$$

by choosing $x_1, \ldots, x_{n-1}$ optimally, with x_0 and x_N given. The method described below is a first order gradient method (Bryson and Ho [1969]) wherein x_i is modified to $x_i + \alpha u_i$ and $\dot{x}_i$ is modified to $\dot{x}_i + \alpha \dot{u}_i$, $i = 0, \ldots, N$. α is a small scalar and $u_0 = u_N = 0$, with u_i being otherwise arbitrary for the moment. Letting

$$J(\alpha) = \delta \sum_{i=1}^{N} f(x_i + \alpha u_i, \dot{x}_i + \alpha \dot{u}_i, i\delta), \qquad (A3)$$

$$f_i^1 = \frac{\partial}{\partial x_i} f(x_i, \dot{x}_i, i\delta), \text{ and} \qquad (A4)$$

$$f_i^2 = \frac{\partial}{\partial \dot{x}_i} f(x_i, \dot{x}_i, i\delta), \qquad (A5)$$

* (A2) is the simplest discrete version of (A1), but it is unsymmetric with respect to time because $\dot{x}_i$ involves x_{i-1} but not x_{i+1}. A symmetric version is actually used in JITTER, but otherwise JITTER's method is as described here.

we have that the derivative of J with respect to α, evaluated at $\alpha = 0$, is

$$J' = \delta\sum_{i=1}^{N}\left(f_i^1 u_i + f_i^2 \dot{u}_i\right). \tag{A6}$$

The symbol $\dot{u}_i$ is being used to symbolize the backward difference $(u_i-u_{i-1})/\delta$; with $\dot{g}$ below similarly being the backward difference of g. Note that f_i^1 and f_i^2 are vectors of the same dimension as x, and that the products in (A6) are inner products.

Let $g_i \equiv \delta\sum_{j=1}^{i} f_j^1$; $i = 0,\ldots,N$, so that $f_i^1 = \dot{g}_i$. Using summation by parts, we have

$$\delta\sum_{i=1}^{N} f_i^1 u_i = \delta\sum_{i=1}^{N}\dot{g}_i u_i = u_N g_N - u_0 g_0 - \delta\sum_{i=1}^{N} g_{i-1}\dot{u}_i. \tag{A7}$$

Using (A7) and the fact that $u_0 = u_N = 0$, (A6) is

$$J' = \delta\sum_{i=1}^{N} h_i \dot{u}_i, \text{ where} \tag{A8}$$

$h_i \equiv f_i^2 - g_{i-1}$. Recall that the quantities u_i are still unspecified, except that $\sum_{i=1}^{N}\dot{u}_i = 0$ because $u_N - \mu_0 = 0$. If J′ is to vanish regardless of how u_i is specified, it follows that h_i must be constant for i = 0, ..., N. (A8) therefore determines the optimal trajectory to within a (vector) constant. Finding the constant that is consistent with the boundary conditions is a two-point-boundary-value problem, with erroneous constants corresponding to trajectories that are optimal but not feasible. However, an iterative method where all trajectories are feasible can also be based on (A8), since for any feasible trajectory the (generally nonconstant) quantities h_i determine an improving direction. Specifically, let $\mu_0 = 0$ and

$$\dot{u}_i = \bar{h} - h_i; \quad i = 1,\ldots,N., \text{ where} \tag{A9}$$

$\bar{h} \equiv \frac{1}{N}\sum_{i=1}^{N} h_i$. Subtracting each h_i from the average $\overline{h}$ forces u_N to be 0, as required. Substituting (A9) into (A8), we have

$$J' = \delta \sum_{i=1}^{N} h_i\left(\bar{h} - h_i\right) = \delta N Q, \text{ where} \tag{A10}$$

$Q = \bar{h}\bar{h} - \frac{1}{N}\sum_{i=1}^{N} h_i h_i$. Since $Q \le 0$, with equality possible only when h_i is constant (that is, h_i is independent of i), (A10) results in a negative value for J′ unless the trajectory is already locally optimal.

An all feasible gradient method is now clear:

1) guess a feasible trajectory x_i; $i = 0, \ldots, N$

2) let $\dot{x}_i = (x_i - x_{i-1}) / \delta; i = 1,\ldots,N$

3) use (A9) to determine $\dot{u}_i; i = 1,\ldots,N$

4) let $u_i = \sum_{j=1}^{i} \dot{u}_j; i = 1,\ldots,N-1$

5) let $x_i' = x_i + \alpha u_i; i = 1,\ldots,N-1$

6) replace x_i with x_i' and go to 2).

The only remaining issues are the usual ones in first order methods: determination of the step size α and the stopping criterion. Since αJ′ is a forecast of the decrease to be expected with each modification, a natural dynamic step size method would increase (decrease) α if the forecast

improvement is (is not) approximately equal to the actual improvement. This is the method used in JITTER.

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